

# Partition Functions of Heterotic Potentials

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July 7, String Phenomenology 2022  
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# I: Introduction and Motivation

Physics / Phenomenon:

- The world is Quantum!

Mathematics:

- TQFT  $\leadsto$  geometric / enumerative / topological invariant theory.

Heterotic: All sectors are coupled.

- This talk:
- $\alpha' = 0$ ; decouple gauge sector
  - $H^*(X) = 0$ .

## II: "Toy - Toy - Model": Cheon - Scavoas

$$S_{CS}(A) = \int_{M_3} \text{tr} \left( A dA + \frac{2}{3} A^3 \right)$$

$A \in \mathfrak{su}'(g)$  ,  $\dim(M_3) = 3$ .

$$\text{EOM : } F(A_0) = dA_0 + A_0 \wedge A_0 = 0.$$

1-loop Action:  $A = A_0 + d$

$$\Rightarrow S(d) = \int_{M_3} d d_0 d \quad , \quad d_0 = d + A_0 .$$

Partition function :

$$Z(M_3) = \frac{1}{\text{Vol}(G)} \int \mathcal{D}d e^{-S(d)}$$

~~~ Topological invariant of  $M_3$ .

$S(d)$  has gauge symmetry :  $d \rightarrow d + d_0 \epsilon$ .

$\Rightarrow$  Quantise using BV-BRST formalism.

OR: Intuitive Approach [Pestun - Witten '05]

Sufficient for quadratic actions...

Choose metric  $g$  on  $M_3$ :

$\Rightarrow$  Hodge decomposition:  $\Omega' = d_g \Omega^0 \oplus d_g^+ \Omega^2$

$$\Rightarrow Z = \frac{1}{\text{Vol}(G)} \int_{\alpha \in d_g \Omega^0} \int_{\delta \in d_g^+ \Omega^2} D_\alpha \exp(-S(\alpha))$$

$$= \frac{1}{\text{Vol}(G)} \frac{\text{Vol}(d_g \Omega^0)}{\det(d_g^+ : d_g^+ \Omega^2 \rightarrow d_g \Omega^1)^{\frac{1}{2}}}$$

Goal: Write answer in terms of determinants of elliptic operators (Laplacians), whose determinants can be regularised.

Denominator :  $\det(d_o : d_o^+ \mathbb{R}^2 \rightarrow d_o \mathbb{R}^2)^\frac{1}{2}$

We define :

$$\det(d_o |_{d_o^+ \mathbb{R}^2}) := \det(d_o^+ d_o |_{d_o^+ \mathbb{R}^2})^\frac{1}{2}$$

Note :

$$\det(d_o^+ d_o |_{d_o^+ \mathbb{R}^2}) = \frac{\det(d_o^+ d_o |_{d_o^+ \mathbb{R}^2}) \det(d_o d_o^+ |_{d_o \mathbb{R}^o})}{\det(d_o d_o^+ |_{d_o \mathbb{R}^o})}$$

$$= \frac{\det(A')}{\det(d_o d_o^+ |_{d_o \mathbb{R}^o})}$$

Consider eigenvector  $d_o \alpha$  of  $d_o d_o^+$  on  $d_o \mathbb{R}^o$ :

$$d_o d_o^+ d_o \alpha = \lambda d_o \alpha$$



( $d_o$  is invertible on  $\text{Im}(d_o^+)$ )

$$\underbrace{d_o^+ d_o}_{\Delta^o} \alpha = \lambda \alpha$$

$\Rightarrow \det(d_o d_o^+ |_{d_o \mathbb{R}^o}) = \det(\Delta^o)$

$\Rightarrow \det(d_o^+ d_o |_{d_o^+ \mathbb{R}^2}) = \frac{\det(\Delta')}{\det(\Delta^o)}$

Naamoperator:  $\text{Vol}(d_0 \mathbb{S}^0)$ .

Recall: Given Linear operator  $A: V \rightarrow W$

$$\Rightarrow \text{Vol}(AV) = \det(A) \frac{\text{Vol}(V)}{\text{Vol}(\ker(A))}$$

$$\Rightarrow \text{Vol}(d_0 \mathbb{S}^0) = \underbrace{\det(d_0|_{\mathbb{S}^0})}_{\det(\Delta^0)^{\frac{1}{2}}} \frac{\text{Vol}(\mathbb{S}^0)}{\text{Vol}[\underbrace{\ker(d_0|_{\mathbb{S}^0})}_{H^0(M_3)}]} = 1$$

$$\Rightarrow \text{Vol}(d_0 \mathbb{S}^0) = \det(\Delta^0)^{\frac{1}{2}} \text{Vol}(\mathbb{S}^0)$$

Collect :

$$\mathcal{Z}(M_3) = \frac{1}{\cancel{\text{Vol}(\Delta')}} \frac{\det(\Delta^0)^{\frac{3}{q}}}{\det(\Delta')^{\frac{1}{q}}} \cancel{\text{Vol}(\Delta^0)}$$

$$\Rightarrow \mathcal{Z}(M_3) = \frac{\cancel{|}\Delta^0|^{ \frac{3}{q}}}{\cancel{|}\Delta'|^{ \frac{1}{q}}}$$

This is the Ray-Singer torsion of  $M_3$ .

This is Topological !

### III: Heterotic Superpotential

Superpotential [Cardoso et al '03, Garricci et al '04, ...]:

$$W = \int_{X_6} (H + i d\omega) \wedge \Omega$$

For us:

- Background  $X_6$  is Calabi-Yau.
- Fluctuations are generic:

$$d\delta\omega \neq 0, \quad d\delta\Omega \neq 0, \dots$$

=> 1-loop action:

$$S = \int_{X_6} [x \wedge \bar{\partial}x + b \wedge \bar{\partial}x + c \bar{\partial}b]$$

$x \in \Omega^{1,1}$ ,  $\chi \in \Omega^{2,1}$ ,  $b \in \Omega^{0,2}$ ,  $c \in \Omega^{3,0}$ .

$\Rightarrow$  1-loop partition function:

$$Z_W(x_6) = \frac{|\tilde{\Delta}^{1,0}| |\tilde{\Delta}^{0,1}|}{|\tilde{\Delta}^{1,1}|^{\frac{1}{2}} |\tilde{\Delta}^{0,0}|^{\frac{3}{2}}}$$

Again,  $Z_W(x_6)$  is topological!

Can compare with 1-loop partition function of  
Kähler potential of complex structures on  $X_6$ :

$$e^{-K} = i \int_{X_6} \omega \wedge \bar{\omega}$$

This is computed in [Pestun-Witten '05].

We find:

$$\sqrt{Z_W(X_6)} = Z_{S^2}(X_6)$$

Speculation:

Note that the 1-loop partition function of **IIA** and **IIB** agree in the **large volume** limit.

In analogy, we might expect:  $Z_{S^2}(X_6) = Z_W(X_6)$

$$\Rightarrow Z_W(X_6) = Z_{S^2}(X_6) \quad Z_W(X_6) = Z_{\underset{\uparrow}{K}}(X_6) \quad (x)$$

at large volume ( $\alpha' = 0$ ).

het. Kähler potential

## IV: Conclusion / Outlook

- The 1-loop partition function of the heterotic superpotential (of the geometric sector) is Topological.

### Outlook:

- Check ( $x$ ): Compute  $Z_K(x_c)$ . (in progress)
- Turn on  $\alpha'$ : Couplings to gauge sector...  
$$(\text{Het. Bianchi: } dH = \frac{\alpha'}{4} ( f_L F \wedge F - f_R R \wedge R ))$$
- Applications in  $(0,2)$ -Mirror Symmetry?

Thank You !